I/Q diagram of coupled system and logarithmic singularity

Yutong Zhao

Nov 18th 2019

Case 0: Empty Cavity

Transmission of a single resonance

$$S_{21} = 1 - \frac{\kappa_1}{i(\omega - \omega_1) + \kappa + \gamma}$$

Realising the Denominator

$$S_{21} = \frac{[i(\omega - \omega_1) + \gamma_1][i(\omega - \omega_1) + \kappa_1 + \gamma_1]}{[i(\omega - \omega_1) + \kappa_1 + \gamma_1][i(\omega - \omega_1) - \kappa_1 - \gamma_1]}$$

$$= \frac{(\omega - \omega_1)^2 + \gamma_1(\gamma_1 + \kappa_1) + i\kappa_1(\omega - \omega_1)}{(\omega - \omega_1)^2 + (\kappa_1 + \gamma_1)^2}$$

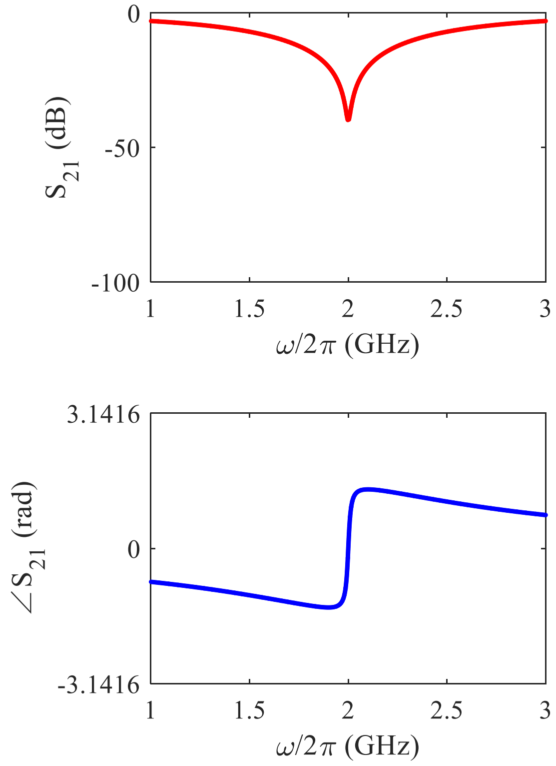
Define the variable

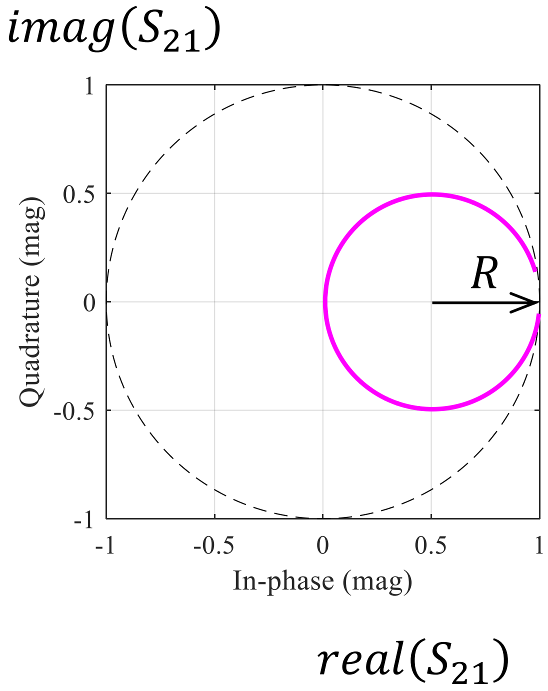
$$X = real(S_{21}) = \frac{(\omega - \omega_1)^2 + \gamma_1(\gamma_1 + \kappa_1)}{(\omega - \omega_1)^2 + (\kappa_1 + \gamma_1)^2}$$

$$Y = imag(S_{21}) = \frac{\kappa_1(\omega - \omega_1)}{(\omega - \omega_1)^2 + (\kappa_1 + \gamma_1)^2}$$

$$(X - X_C)^2 + Y^2 = R^2$$

$$X_C = 1 - \frac{\kappa_1}{2(\kappa_1 + \gamma_1)} \qquad R = \frac{\kappa_1}{2(\kappa_1 + \gamma_1)}$$





Case 1: Coherent coupling ($g \in \mathbb{R}$)

Transmission of a coupled system

$$S_{21} = 1 - \frac{\kappa_1}{i(\omega - \omega_1) + \kappa_1 + \gamma_1 + \dfrac{g^2}{i(\omega - \omega_2) + \kappa_2 + \gamma_2}}$$

Assume that $\omega \approx \omega_2$, near the resonance frequency of 2

$$S_{21} = 1 - \frac{\kappa_1}{i\Delta + \kappa_1 + \gamma_1 + \dfrac{g^2}{i(\omega - \omega_2) + \kappa_2 + \gamma_2}}$$

We set $\Delta = 0$ for simplification

$$S_{21} = 1 - \frac{\kappa_1}{\kappa_1 + \gamma_1} + \frac{1}{(\kappa_1 + \gamma_1)^2} \frac{g^2}{i(\omega - \omega_2) + \kappa_2 + \gamma_2}$$

Constant
(fixed point on circle)

The trajectory of ω_2

Taylor expansion near ω_2

$$\frac{1}{c + x} = \frac{1}{c} - \frac{1}{c^2} x + \frac{1}{c^3} x^2 + \cdots$$

Realising the Denominator

$$\frac{g^2[i(\omega-\omega_2)+\kappa_2+\gamma_2]}{[i(\omega-\omega_2)+\kappa_2+\gamma_2][i(\omega-\omega_2)-\kappa_2-\gamma_2]}$$

Define expanded variables

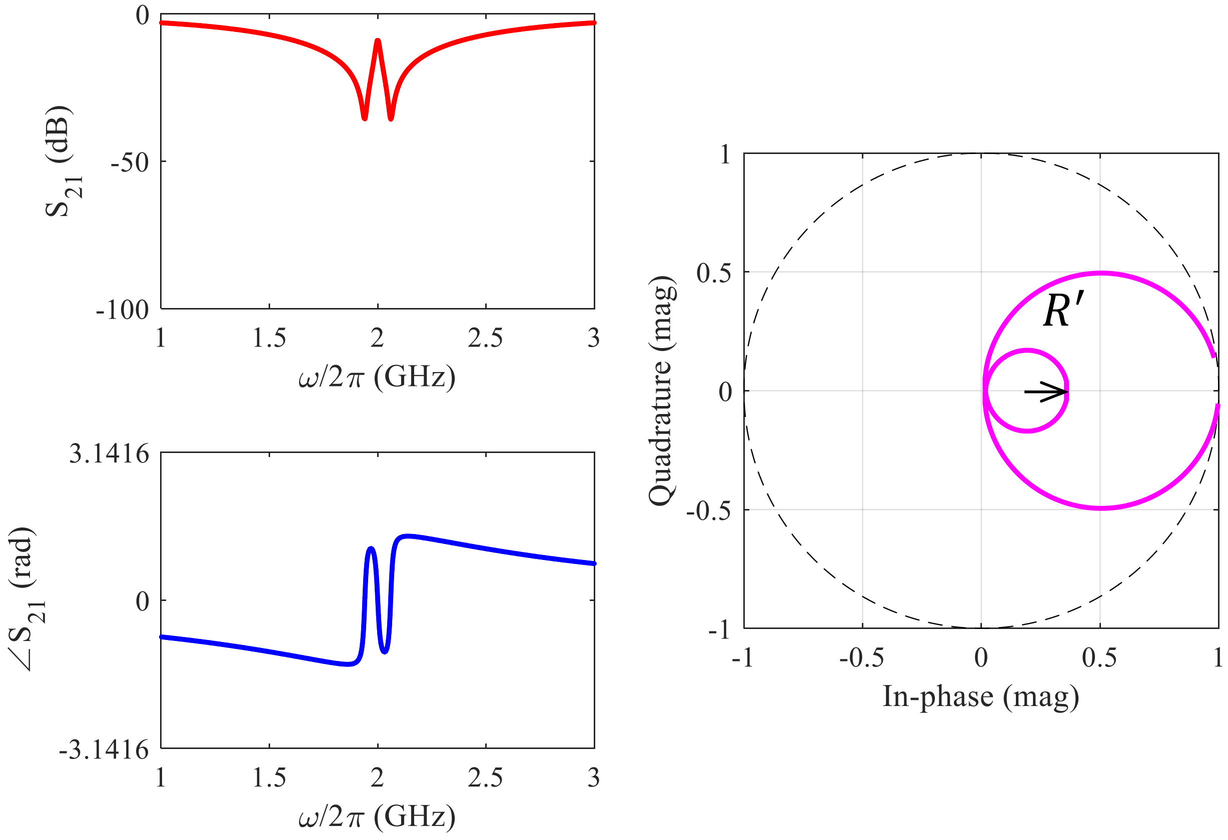
$$X' = \frac{g^2(\kappa_2+\gamma_2)}{(\omega-\omega_2)^2+(\kappa_2+\gamma_2)^2}$$

$$Y' = \frac{g^2(\omega-\omega_2)}{(\omega-\omega_2)^2+(\kappa_2+\gamma_2)^2}$$

$$X'^2 + Y'^2 = R'^2$$

$$R' = g^2/(\kappa_1+\gamma_1)^2$$

$$X_{c^2} = 1 - \frac{\kappa_1(\kappa_1+\gamma_1)-g^2}{(\kappa_1+\gamma_1)^2}$$



X_{c^2} inside circle 1 → Incircle

Case 2: Dissipative coupling ($g \in \mathbb{I}$)

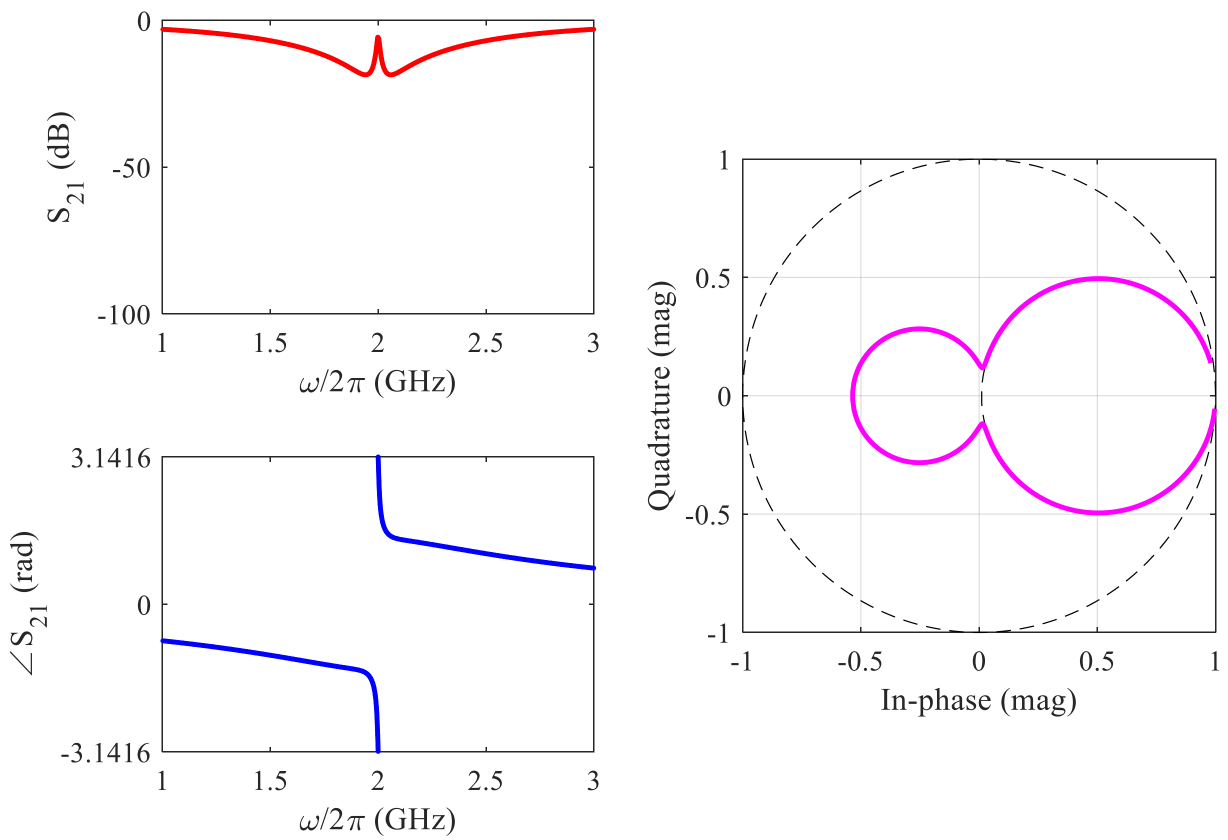
Expanded variables

$$X' = -\frac{|g|^2(\kappa_2 + \gamma_2)}{(\omega - \omega_2)^2 + (\kappa_2 + \gamma_2)^2}$$

$$Y' = -\frac{|g|^2(\omega - \omega_2)}{(\omega - \omega_2)^2 + (\kappa_2 + \gamma_2)^2}$$

$$X'^2 + Y'^2 = R'^2$$

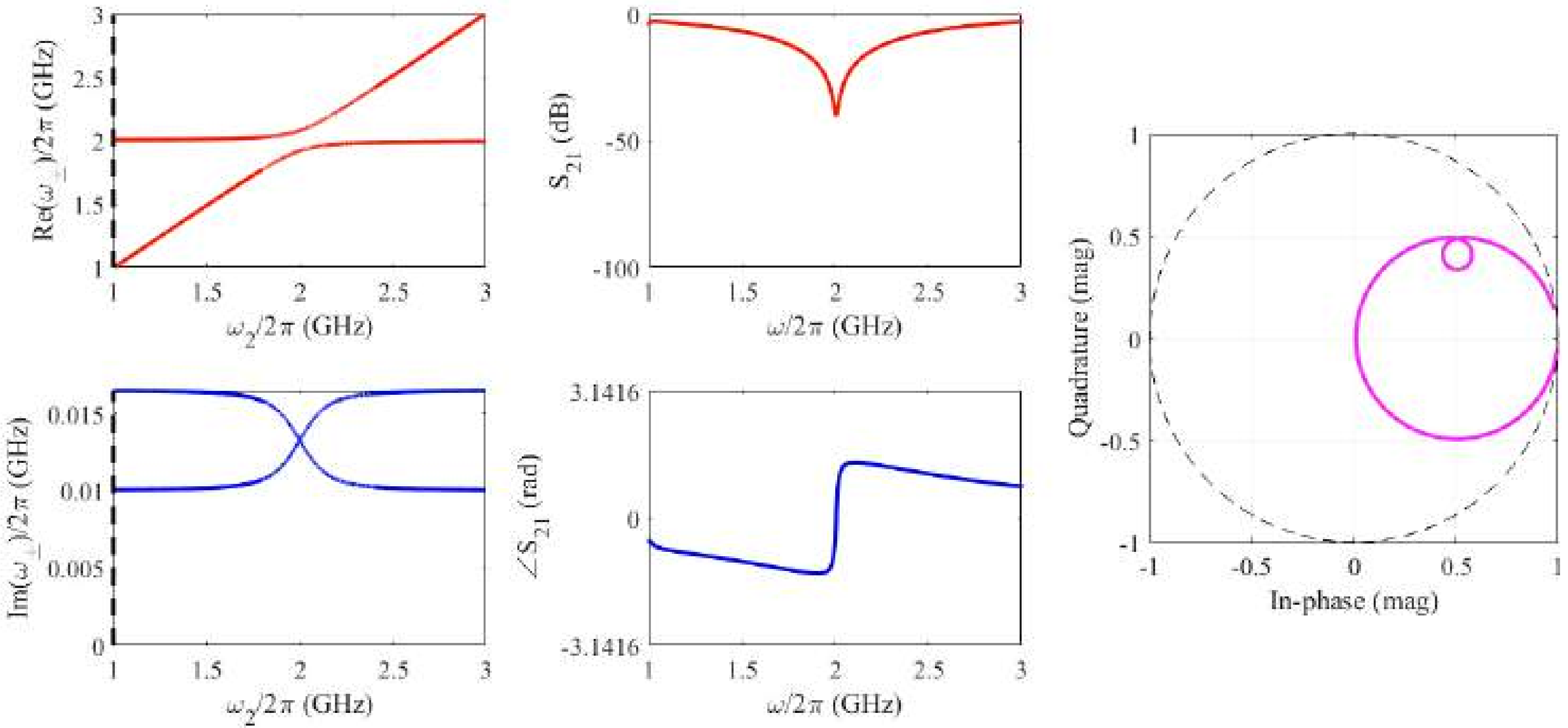
$$R' = -|g|^2/(\kappa_1 + \gamma_1)^2$$



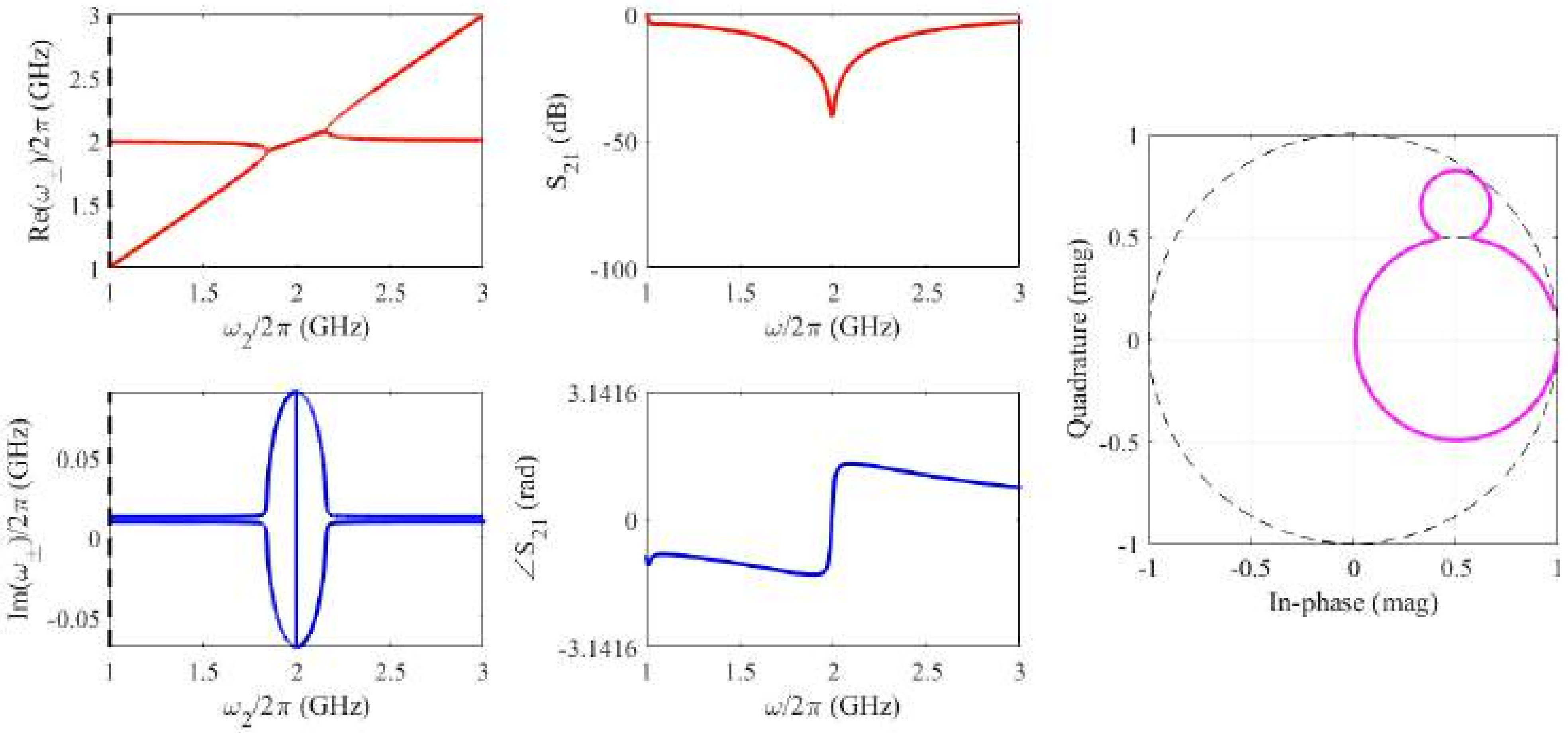
$$X_{c^2} = 1 - \frac{g^2 + \kappa_1(\kappa_1 + \gamma_1)}{(\kappa_1 + \gamma_1)^2}$$

X_{c^2} outside circle 1 → Excircle

Case 1: Coherent coupling ($g \in \mathbb{R}$)



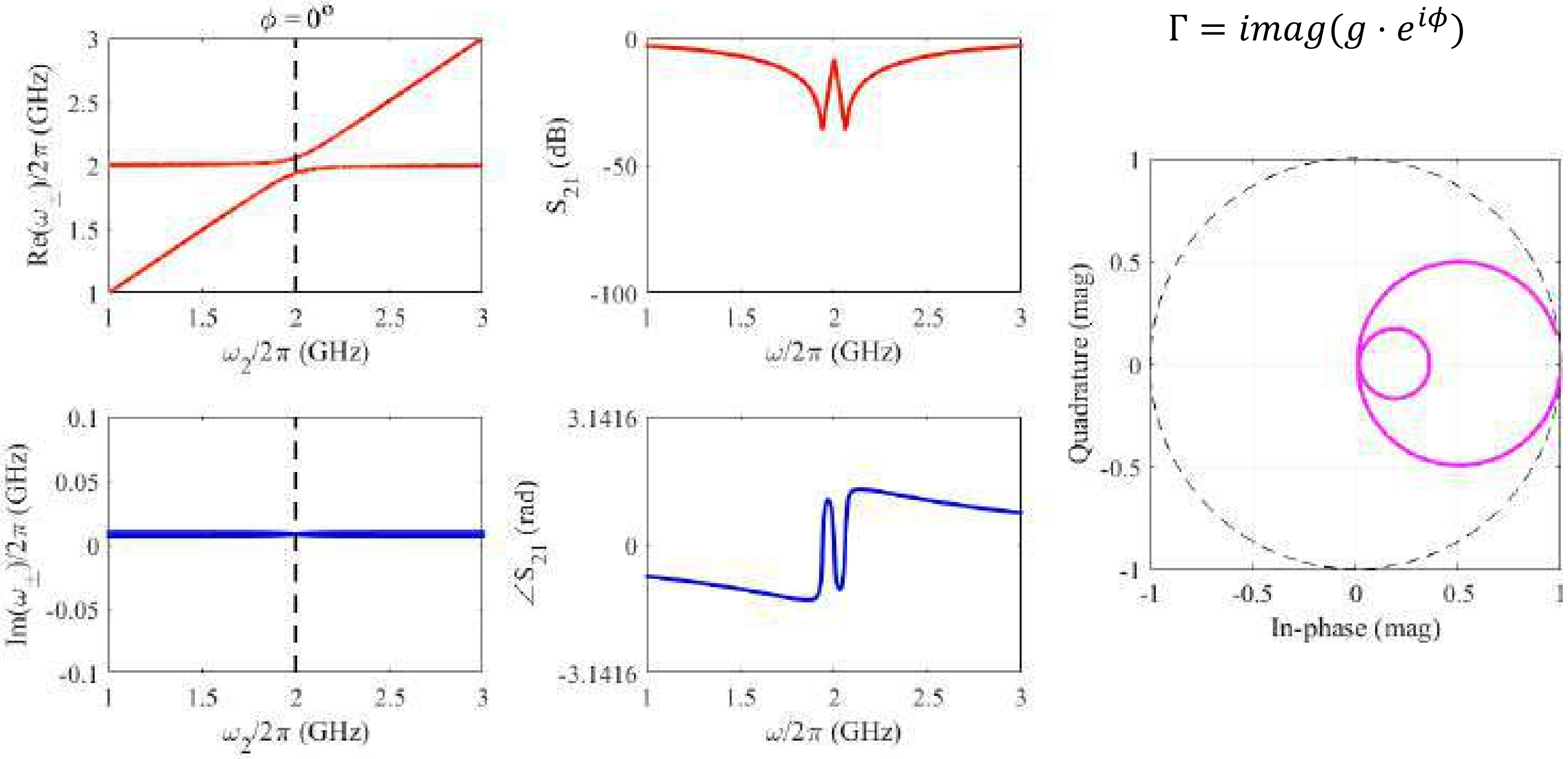
Case 2: Dissipative coupling ($g \in \mathbb{I}$)



Case 3: From Coherent to Dissipative coupling ($g \in \mathbb{I} \rightarrow \mathbb{R}$ at $\Delta = 0$)

$J = real(g \cdot e^{i\phi})$

$\Gamma = imag(g \cdot e^{i\phi})$



nature materials | REVIEW ARTICLE
PUBLISHED ONLINE: 23 AUGUST 2010 | DOI: 10.1038/NMAT2810

The Fano resonance in plasmonic nanostructures and metamaterials

Boris Luk'yanchuk[1], Nikolay I. Zheludev[2], Stefan A. Maier[3], Naomi J. Halas[4], Peter Nordlander[5*], Harald Giessen[6] and Chong Tow Chong[1,7]

the classical formula for Rayleigh scattering:

Scattering efficiency $$Q_{sca} \approx \frac{8}{3}\left|\frac{\varepsilon - 1}{\varepsilon + 2}\right|^2 q^4$$

This formula holds for positive and negative ε except close to the surface plasmon resonance ($\varepsilon = -2$), where it exhibits a singularity in the absence of intrinsic damping. With increasing size, higher-

Grigoriev, V., et al. "Singular analysis of Fano resonances in plasmonic nanostructures." *Physical Review A* 88.6 (2013): 063805.

The complex function of S_{21} (transmission in log scale) is everywhere analytical except at the ZDCs.

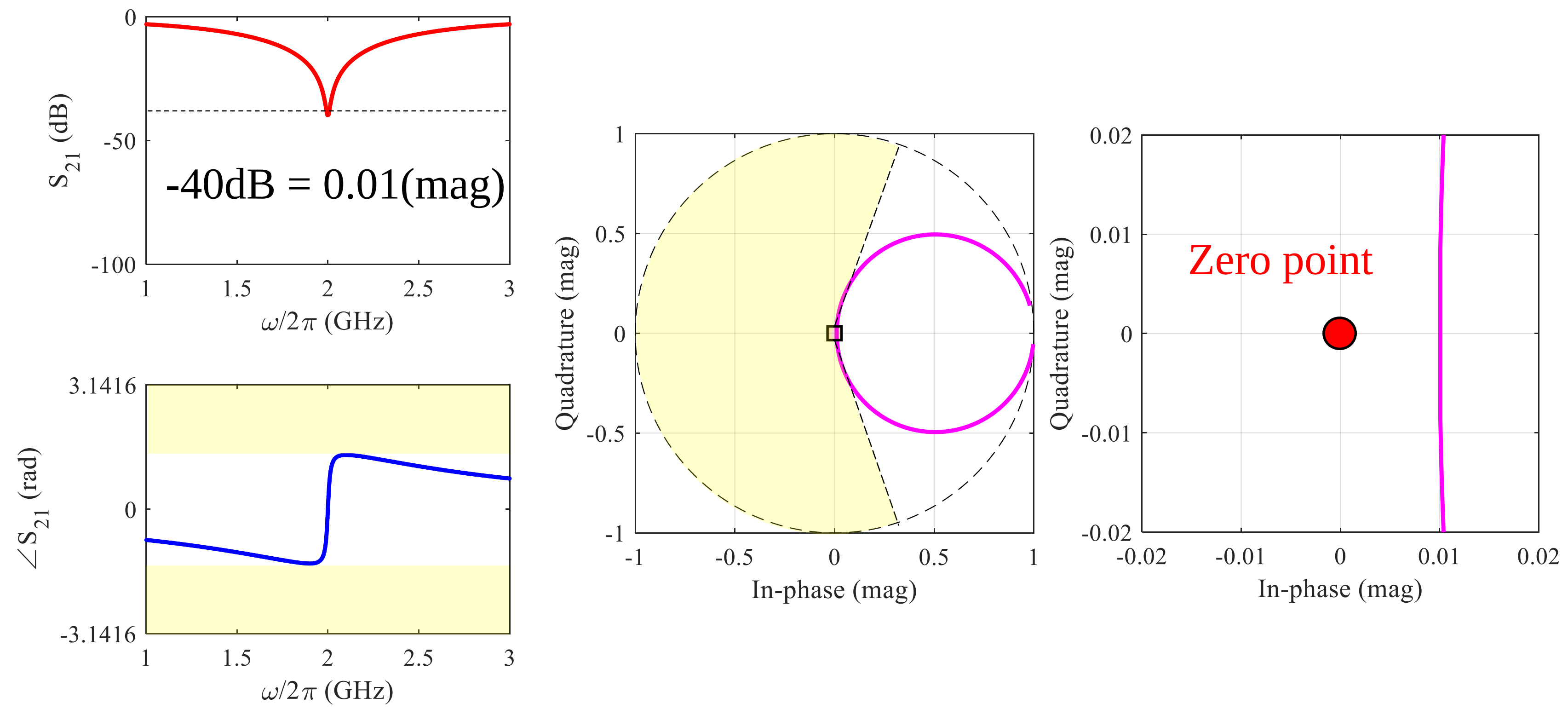
→ logarithmic singularity when $S_{21} = 0$

$$\lim_{\omega \to \omega_{ZDC}} \log_{10}(|S_{21}|) = -\infty$$

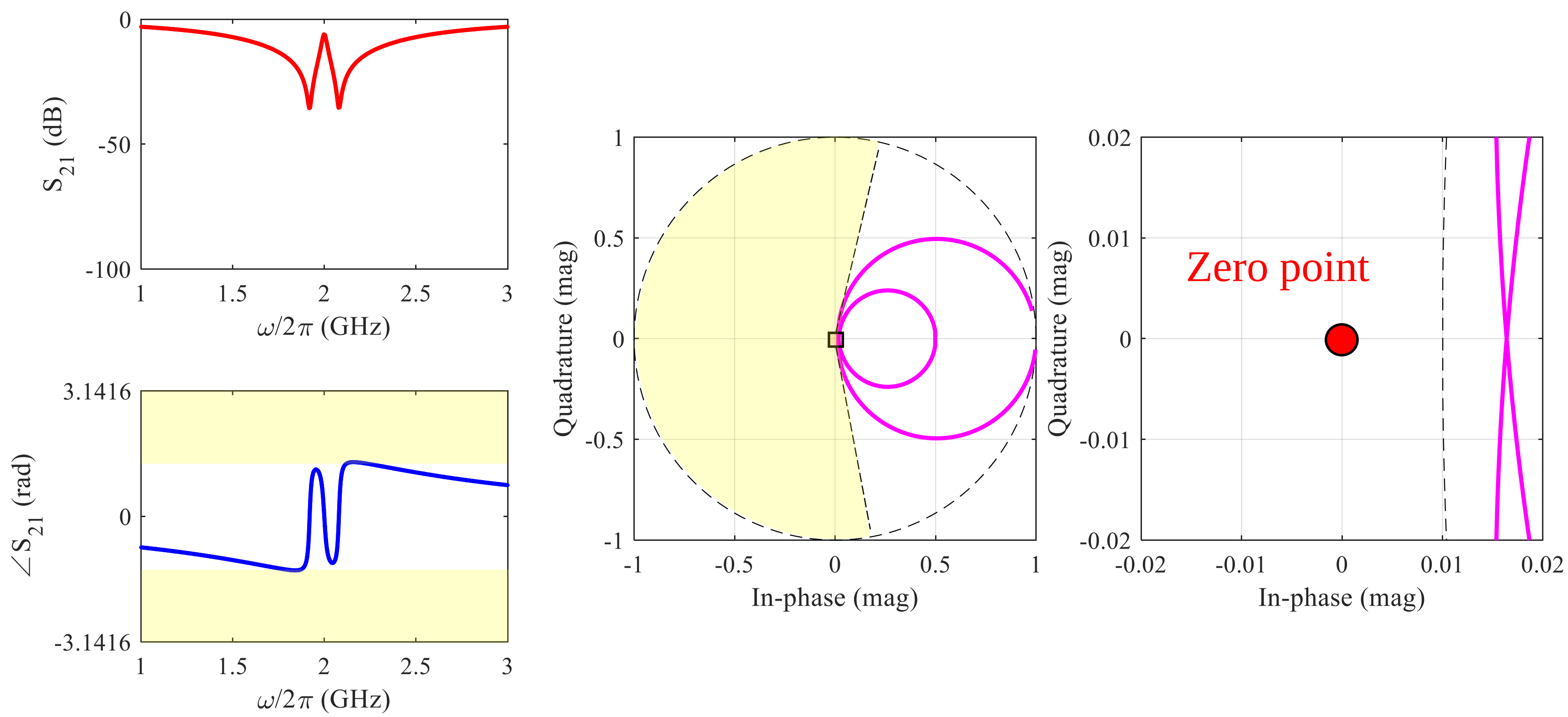
$$\lim_{\omega \to \omega_{ZDC}} \frac{d\phi}{d\omega} = \infty$$

Markushevich, A. I. (1977). *Theory of functions of a complex variable*. American Mathematical Soc..

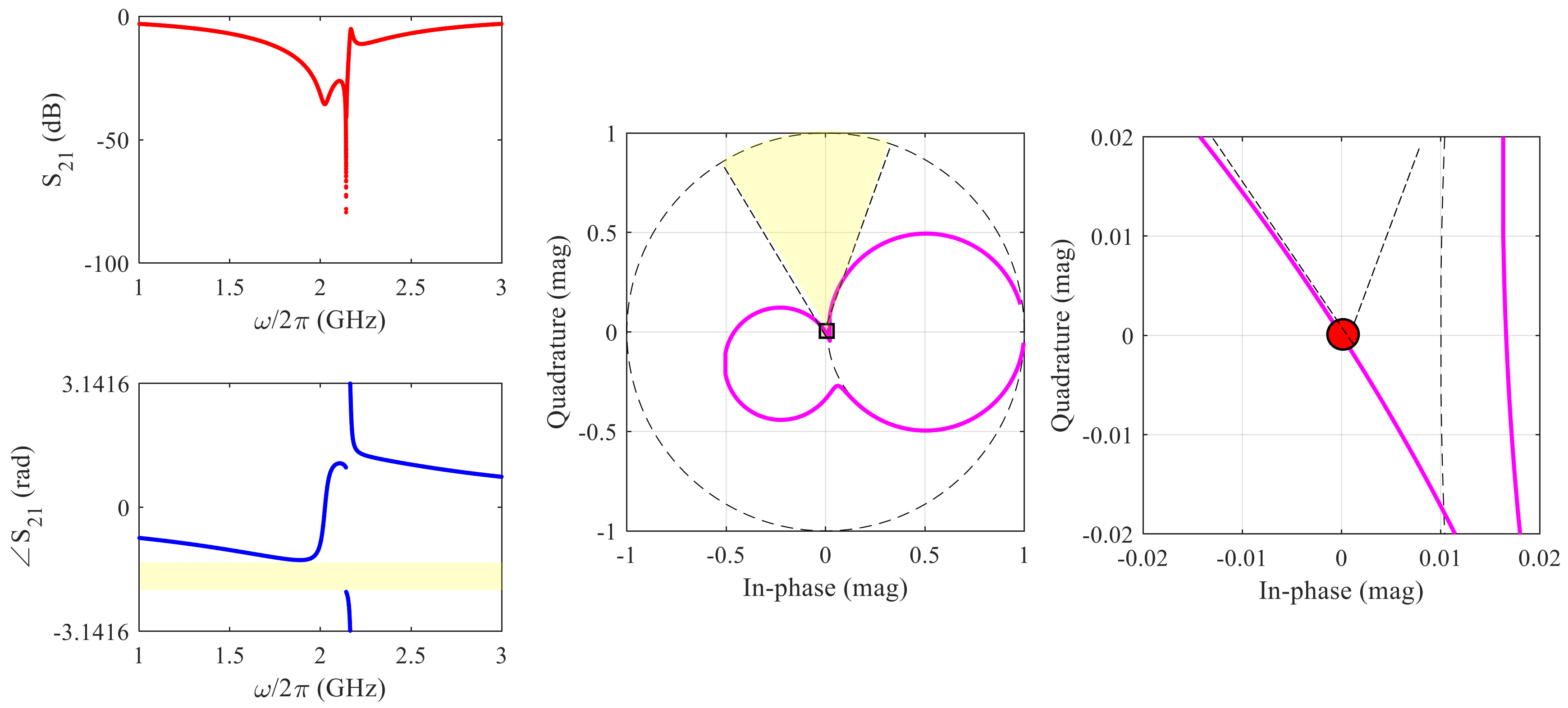
Case 0: Empty Cavity



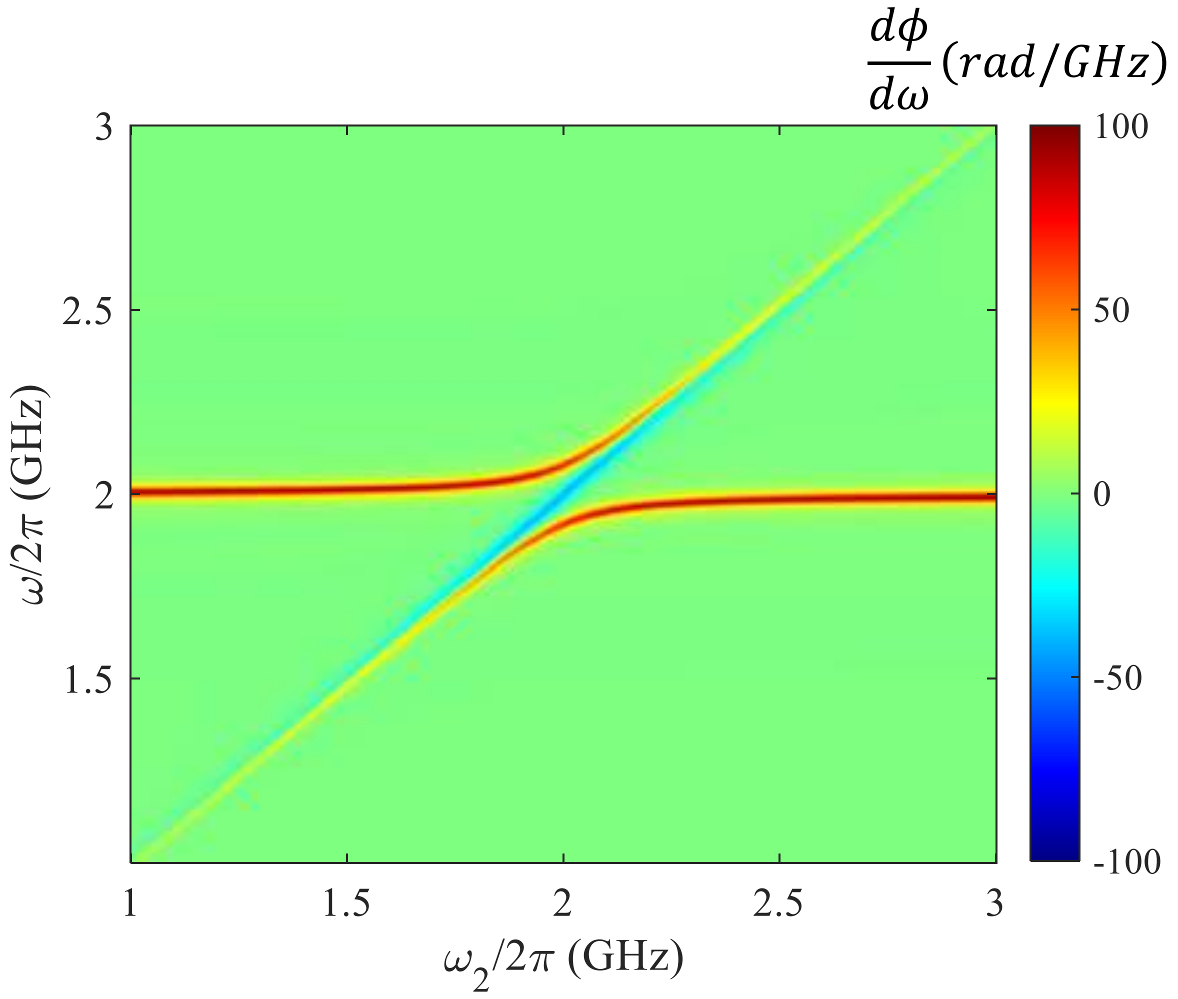
Case 1: Coherent coupling ($g \in \mathbb{R}$)



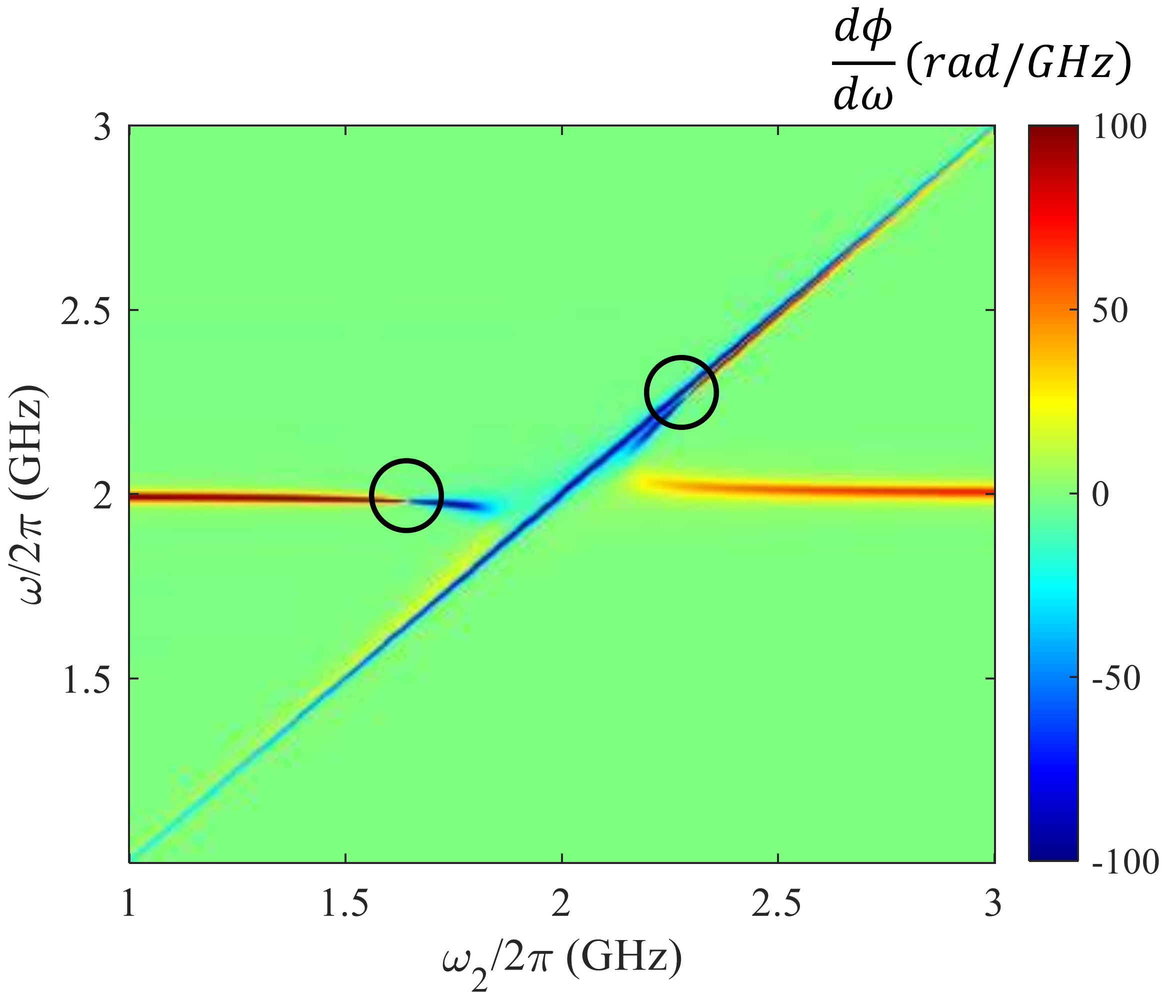
Case 2: Dissipative coupling ($g \in \mathbb{I}$)



Case 1: Coherent coupling ($g \in \mathbb{R}$)



Case 2: Dissipative coupling ($g \in \mathbb{I}$)



Conclusion

- Coherent coupling is limited by the original circle of resonator 1.
- Dissipative coupling breaks the limitation of the coherent coupling and can reach logarithmic singular point that produce ZDC.
- The two π phase jump come from the I/Q curve includes the origin.

Equivalent Q factor of hybridized modes

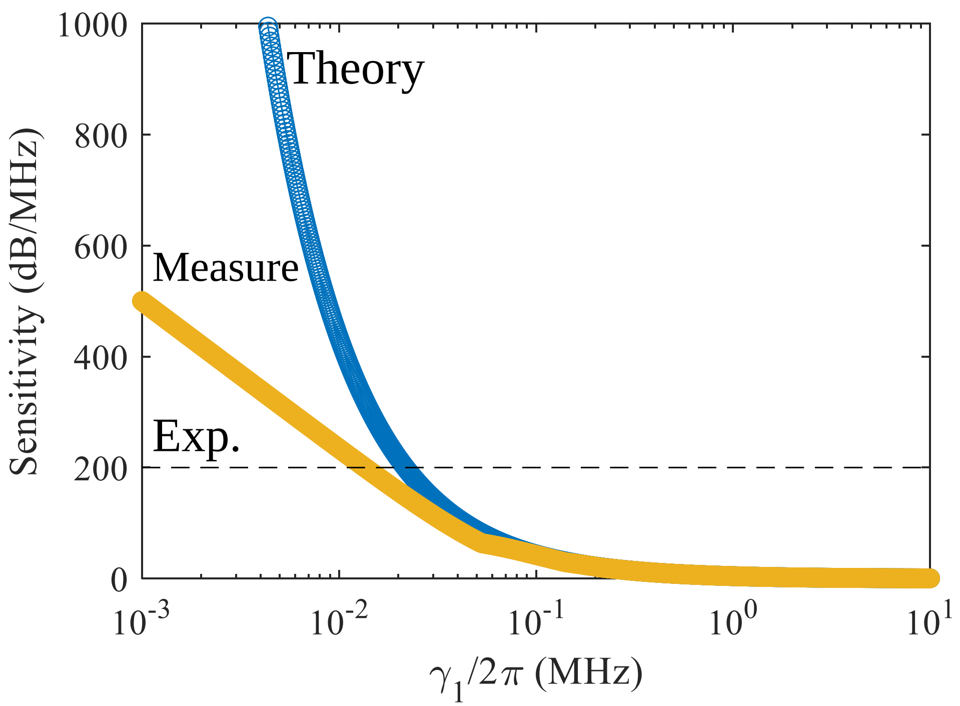
$$S_{21} = 1 - \frac{\kappa}{i(\omega_c - \omega) + \kappa + \gamma}$$

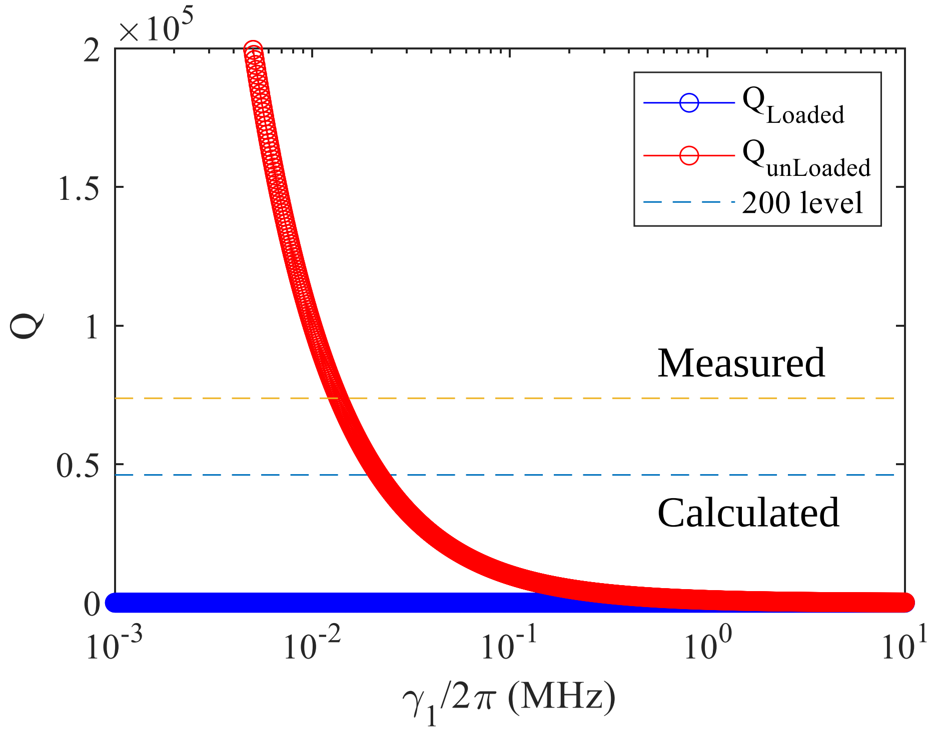
$$\frac{d|S_{21}|}{d\omega}(\gamma) = \frac{d|S_{21}|}{d|S_{21}|^2}\frac{d|S_{21}|^2}{d\omega}$$

Numerical result:

$$Q_{Loaded} \sim \kappa$$
$$Q_{unLoaded} \sim \gamma$$

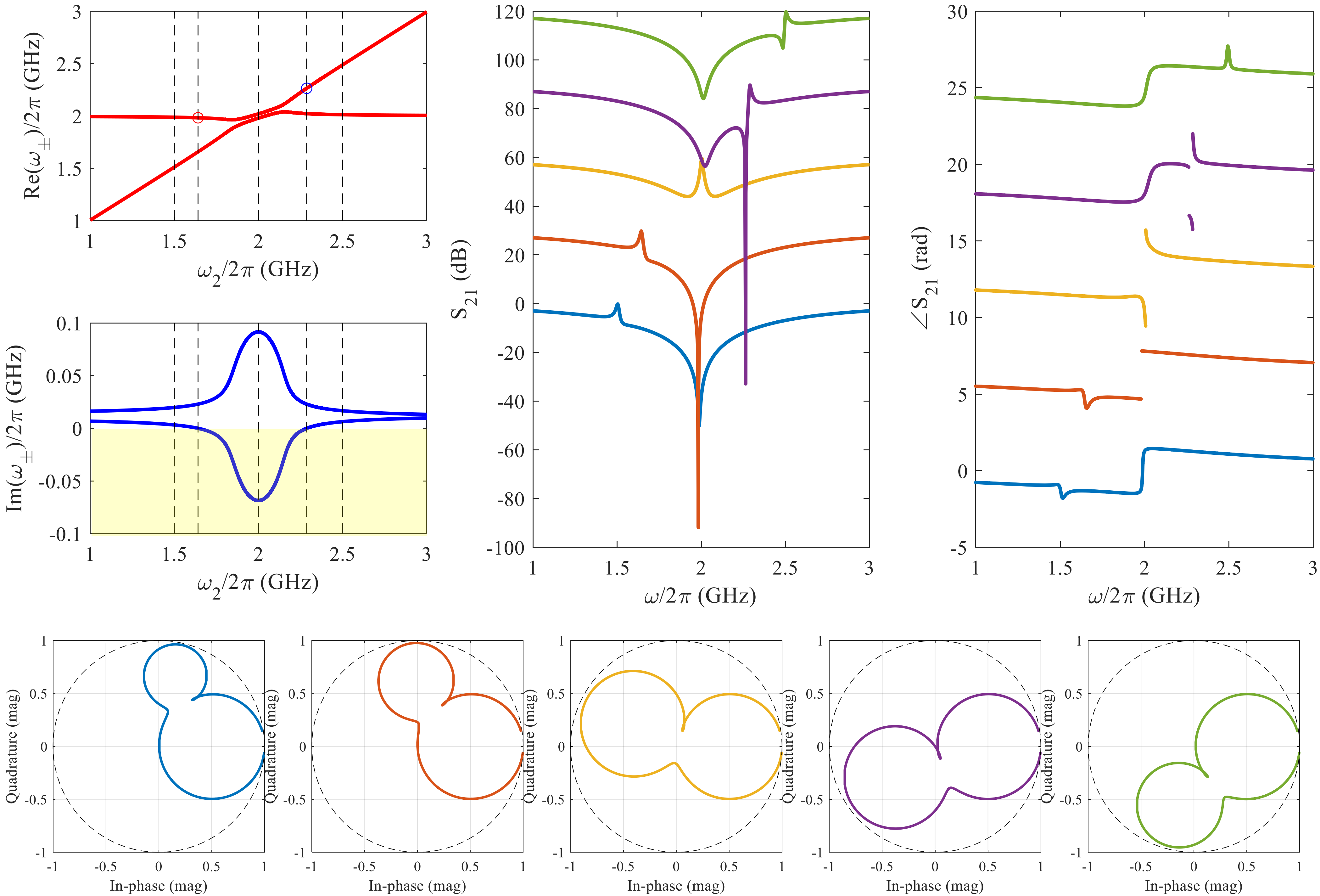
For S = 200 dB/MHz, the equivalent Q_u = 46,000





Q_u = 73,800

Q_u = 46,000



Re($\omega_\pm$)/2π (GHz)
ω_2/2π (GHz)
Im($\omega_\pm$)/2π (GHz)
ω_2/2π (GHz)
S_{21} (dB)
ω/2π (GHz)
$\angle S_{21}$ (rad)
ω/2π (GHz)
Quadrature (mag)
In-phase (mag)